### Tuesday June 13

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<td>Javier Etayo</td>
<td><em>The full group of automorphisms of non-orientable unbordered Klein surfaces</em></td>
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Abstracts for Tuesday June 13

Klein foams as families of real forms of Riemann surfaces
Sergey M. Natanzon
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Klein foams are analogues of Riemann surfaces for surfaces with one-dimensional singularities. They first appeared in mathematical physics (string theory etc.). By definition a Klein foam is constructed from Klein surfaces by gluing segments on their boundaries. We show that, a Klein foam is equivalent to a family of real forms of a complex algebraic curve with some structures. This correspondence reduces investigations of Klein foams to investigations of real forms of Riemann surfaces. We use known properties of real forms of Riemann surfaces to describe some topological and analytic properties of Klein foams. The talk is based on a joint work with Sabir M. Gusein-Zade.

The full group of automorphisms of non-orientable unbordered Klein surfaces
Javier Etayo
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Determining the full automorphism group of Riemann and Klein surfaces is an important problem in this field. There are results known for surfaces of low genus or families of surfaces with special properties, what represents very partial results. In this communication we deal with non-orientable unbordered Klein surfaces. In this case the solution of the problem is known for surfaces of genus 1, 2, 3, 4, 5 and for hyperelliptic surfaces. We will explicitly obtain the full automorphism group of all surfaces of genus 6 and 7. These are results by Adrián Bacelo.

Lifting the hyperelliptic involution of a Klein surface
Peter Turbek
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We consider unbranched normal coverings $X \to X'$ between compact Klein surfaces of algebraic genus bigger than one where $X'$ is hyperelliptic. Here unbranched means that the fixed point set of the group of covering transformations is either empty or projects onto the boundary of $X'$. We find a criterion which determines whether the hyperelliptic involution of $X'$ lifts to an automorphism of $X$. The study splits naturally into six cases according to the different topological types that $X'$ may possess. If the group of covering transformations is abelian and has odd order, we prove that the hyperelliptic involution always lifts. This is a joint work with E. Bujalance and J. Cirre.
Equations, hyperellipticity degree and topological types for the Riemann surface having three symmetries with maximal total number of ovals

Ewa Kozłowska-Walania
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It is well known that three nonconjugate symmetries of a Riemann surface of genus \( g \) have at most \( 2g + 4 \) or \( 2g + 3 \) ovals in total, for \( g \) odd or even respectively. Both these bounds are sharp for any \( g \) of given parity and the group of automorphisms is the elementary abelian group \( \mathbb{Z}_2^3 \). In such a case we describe all the possible distributions of topological types of the three symmetries in question which are realized, and discuss the hyperellipticity degree of the surface. Finally we give equations for the underlying complex curve and its real forms.

Topological rigidity of finite cyclic group actions on bordered surfaces

Blazej Szepietowski
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Two actions of a group on a surface are called topologically equivalent if they are conjugate by a homeomorphism of the surface. I will describe a method of enumeration (and classification) of topological equivalence classes of actions of a finite group on a compact surface, based on the combinatorial theory of noneuclidean crystallographic groups (NEC groups in short) and a relationship between the outer automorphism group of an NEC group and certain mapping class group. By this method we study topological equivalence of actions of a finite cyclic group on a bordered surface, in the situation where the order of the group is large relative to the algebraic genus of the surface. This is joint work with Grzegorz Gromadzki and Susumu Hirose.

A SAGE package for equisymmetric stratification and applications

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In this talk we present an algorithm running over SAGE [8], which allows users to compute group actions on Riemann surfaces up to topological equivalence, in terms of generating vectors. Moreover, it allows to compute induced generating vectors when considering subgroups of a given group, hence to classify the corresponding action up to topological equivalence.

The main motivation to develop this algorithm is to study the equisymmetric stratification \( \mathcal{M}_g = \cup \mathcal{M}(g, G, \theta) \) of the moduli space of Riemann surfaces of genus \( g \) defined in [2], where \( \mathcal{M}(g, G, \theta) \) is the equisymmetric stratum of curves \( X \) of genus \( g \) with a \( G \)–action and epimorphism \( \theta : \Delta \to G \) determining an action \( \sigma \) of \( G \) on \( X \). Here \( \Delta \) is the group uniformizing \( X/G \) with canonical presentation given by

\[
\Delta = \langle \alpha_1, \beta_1, \ldots, \alpha_\gamma, \beta_\gamma, x_1, \ldots, x_r : x_1^{m_1} = \cdots = x_r^{m_r} = \prod_{j=1}^{\gamma} [\alpha_j, \beta_j] \prod_{i=1}^r x_i = 1 \rangle.
\] (1)
Two actions $\sigma_1, \sigma_2$ are topologically equivalent if and only if the epimorphisms $\theta_1, \theta_2$ lie in the same $\mathcal{B} \times \text{Aut}(G)$ class, where $\mathcal{B}$ is the subgroup of $\text{Aut}(\Delta)$ induced by orientation preserving homeomorphisms. Therefore, $\mathcal{M}(g, G, \theta)$ is the stratum of surfaces of genus $g$ with automorphism group $G$ in the conjugacy class in the mapping class group of the action determined by the epimorphism $\theta$. $\mathcal{M}(g, G, \theta)$ denotes the subset of surfaces having an automorphism group containing the automorphism group $G$ in the conjugacy class determined by $\theta$ in the mapping class group $\text{Mod}(\Gamma)$. For references see [2, 4]. In this context, our algorithm is useful to understand how these strata, or their boundaries, intersect in the corresponding moduli space.

Our algorithm works on any genus except for obvious hardware constrains, in the case the genus of the quotient is zero. It is programmed as an update of the algorithm in [1], and as applications we computed all group actions up to topological equivalence for genus 5 to 10, completing the lists in [5, 7]. Besides, we added an improved version of an algorithm given in [3], which allows us to identify Jacobian varieties of CM-type. As a byproduct, we obtain a Jacobian variety of dimension 11 which is isogenous to $E_i^9 \times E_{i\sqrt{3}}^2$, where $E_i$ and $E_{i\sqrt{3}}$ are elliptic curves with complex multiplication.

Final remarks, there are other algorithms for group actions on the literature, for instance [6]. The main difference between our algorithm and [6] is that the last one computes actions fixing the ramification type while we fix the signature hence we obtain all possible ramification types, moreover we classify them up to topologically equivalence, and study the actions of subgroups. This is joint work with A. Behn and M. Tello.

Partially supported by Fondecyt Grant 1140507 and CONICYT PIA ACT1415.

Relations between real algebraic surfaces and complex curves
Santiago López de Medrano
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In [1] we presented some families of real affine surfaces which are transverse intersections of concentric ellipsoids and some remarkable coincidences between their genera and those of certain natural families of smooth algebraic curves.

We now part from this relation to give many examples of complex projective varieties with dihedral symmetry. The varieties are constructed using rows of the Vandermonde matrix on the n-th roots of unity (also known as the Discrete Fourier Transform matrix). Applying some results about these minors to the case of curves we prove that any Riemann surface which is a complete intersection of hypersurfaces has dihedral symmetry.
This is joint work with Matthias Franz and Vinicio Gómez Gutiérrez.


Weierstrass gaps and graph coverings
Klara Stokes
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Coverings of curves are useful for calculating the Weierstrass gaps of pointed curves. In this talk I will consider analogies of Weierstrass gaps for graphs, and use coverings of graphs for calculations. This is joint work with Milagros Izquierdo.

On (q, n)-gonal pseudo-real Riemann surfaces
Ewa Tyszkowska
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A compact Riemann surface $X$ of genus $g \geq 2$ is called $p$-hyperelliptic, if it admits a conformal involution $\rho$ such that the orbit space $X/\langle \rho \rangle$ is an orbifold of genus $p$. A $p$-hyperelliptic surface is the particular case of so-called cyclic $(q, n)$-gonal surface, which is defined as the one admitting a conformal automorphism $\delta$ of prime order $n$ such that $X/\langle \delta \rangle$ has genus $q$. We say that a Riemann surface is pseudo-real if it has an anticonformal automorphism but no anticonformal involution. An anticonformal automorphism of such a surface has order divisible by 4, and is called a pseudo-symmetry if its order is equal to 4. For given integers $g \geq 2$ and $k \geq 1$, we give the necessary and sufficient conditions for the existence of a pseudo-real Riemann surface $X$ of genus $g$ whose full automorphism group is a cyclic group $\mathbb{Z}_{4k}$, and we determine $(q, n)$-gonal automorphisms of the surface. We prove that for $g \geq 12$ and every integer $p$ in the range $0 \leq p \leq (g+1)/2$ having the same parity as $g$, there exists a $p$-hyperelliptic pseudo-real Riemann surface of genus $g$ which may not be pseudo-symmetric.
The homological invariant for effective group actions on compact Riemann surfaces
Michael Lönne
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Effective group actions on compact Riemann surfaces give rise to Nielsen invariants, which collect the local monodromies for the corresponding quotient map. They have been shown by Conway and Parker to determine the group action in cases of “large” Nielsen invariants and groups satisfying a certain homological condition. In the talk the homological invariant allowing a classification for arbitrary finite groups will be explained and its applications will be given. (Joint work with Fabrizio Catanese and Fabio Perroni).

Loci of curves with dihedral symmetry
Fabio Perroni
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Riemann surfaces with a finite cyclic group of symmetries have been studied by many mathematicians, motivated by several different applications. The corresponding moduli spaces are important (for example) in the study of the singular locus of the moduli space of curves, as shown in works of Cornalba and Catanese.

In the seminar I will report on a joint work with Fabrizio Catanese and Michael Lönne, where we investigate the geometry of the moduli space of smooth algebraic curves (compact Riemann surfaces) with an effective action of a dihedral group $D_n$. Using a new homological invariant (that we defined for effective actions of any finite group on smooth curves) we first classify the components of this moduli space (these correspond to the topological types of the dihedral group actions on compact Riemann surfaces). We then use this result to describe the components of the locus (in the moduli space) of smooth curves admitting an effective action by $D_n$ (the complete classification of these components has been achieved recently by Binru Li and Sascha Weigl).
A generalization of the Recillas trigonal construction
Rubí E. Rodríguez
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The Recillas trigonal construction gives a bijection between Jacobians of tetragonal curves and Prym varieties of double covers of trigonal curves, under appropriate conditions on the corresponding covers. In this talk we will show that this is the first case, for \( p = 3 \), of a more general construction: under appropriate conditions, for every odd prime number \( p \), there is a bijection between Jacobians of \( 2p^{-1} \)-gonal curves and Prym varieties of double covers of \( p \)-gonal curves. This is a joint work with Ángel Carocca.

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Jacobian varieties with group action
Sebastián Reyes-Carocca
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In this talk we present a generalisation of a theorem due to Kani and Rosen on decomposition of Jacobian varieties of Riemann surfaces with group action. This generalisation extends the set of Jacobians for which it is possible to obtain an isogeny decomposition where all the factors are Jacobians. This is a joint work with Rubí E. Rodríguez.

Partially supported by Postdoctoral Fondecyt Grant 3160002 and Anillo ACT1415 PIA-Conicyt Grant.

Remark about analytic representation
Mariela Carvacho Bustamante
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Given a group \( G \) acting on a compact Riemann surface \( X \), it induces an action on the space of holomorphic 1-forms of \( X \). This action induces a representation group \( \rho \) called analytic representation. What is the relation between two actions topologically equivalent and their respective analytic representations?

The answer of this question is related with the problem to compute the conjugacy classes in \( GL(g, \mathbb{C}) \), where \( g \) is the genus of the surface. It is known that the map to associate the pair \([X,G] \) to \([\rho] \) is injective for \( g = 2 \) and \( 3 \) but for genus \( 4 \) it is not injective [1]. In this talk we show partial results about this problem. This is a work in progress with Víctor González–Aguilera.

Limit points of the branch locus of $\mathcal{M}_g$
Raquel Díaz
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Let $\mathcal{M}_g$ be the moduli space of compact connected hyperbolic surfaces of genus $g \geq 2$. Let $\widetilde{\mathcal{M}}_g$ be the augmented moduli space, which is a compactification of $\mathcal{M}_g$ by adding hyperbolic surfaces with nodes. Let $\mathcal{B}_g \subset \mathcal{M}_g$ the branch locus of $\mathcal{M}_g$, i.e., the subset of hyperbolic surfaces admitting some non-trivial isometry.

The branch locus is stratified by equisymmetric strata, where a stratum consists of hyperbolic surfaces with equivalent action of their preserving orientation isometry group. Any stratum can be described by a certain epimorphism $\Phi$. In this talk, for any of these strata, we describe the topological type of its limits points in $\widetilde{\mathcal{M}}_g$ in terms of $\Phi$. We apply this result to different examples. This is joint work with Víctor González-Aguilera, UTFSM, Chile.

One dimensional strata in the branch locus of moduli space
Antonio F. Costa
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The branch locus of the moduli space of surfaces of genus $g > 2$, is the subspace of surfaces with non-trivial automorphism group. The branch locus admits a stratification by irreducible complex algebraic varieties consisting of surfaces whose automorphism groups have topologically equivalent actions. We describe explicitly those strata of dimension one, as punctured Riemann surfaces, in terms of the action data of the automorphism group. This is a joint work with S. A. Broughton and M. Izquierdo.