Meeting of the Swedish, Spanish and Catalan Mathematical Societies
Umeå, June 12 – June 15, 2017
Special session: Non-Commutative Algebra
Session Program

Tuesday June 13

Time	Speaker	Talk title
14.00–14.30		**Session get-together**
14.30–15.30	Olivier Mathieu	*Self-similarity for τ-groups*
15.30–16.00		**COFFEE BREAK**
16.00–17.00	Pere Ara	*Leavitt path algebras and graph C*-algebras associated to separated graphs*
17.00–17.15		**Break**
17.15–18.15	Mercedes Siles	*Decomposability of Leavitt path algebras*

Wednesday June 14

Time	Speaker	Talk title
14.00–15.00	Ernst Dieterich	*On the classification of four-dimensional unital division algebras*
15.00–15.30	Gustav Hammarhjelm	*On the classification of four-dimensional unital rational division algebras*
15.30–16.00		**COFFEE BREAK**
16.00–17.00	Fernando Montaner	*Local orders and Lesieur Croisot elements in Jordan pairs*
17.00–18.00	Arturo Pianzola	*Lie algebroids arising from infinite dimensional Lie theory*

Thursday June 15

Time	Speaker	Talk title
14.00–15.00	Vladimir Tkachev	*Nonassociative algebras of cubic forms*
15.00–15.30	Ramón González	*Fundamental theorems of Doi-Hopf modules in a non-associative setting*
15.30–16.00		**COFFEE BREAK**
16.00–17.00	Consuelo Martínez	*Graded and conformal modules over superconformal algebras*
17.00–17.15		**Break**
17.15–18.15	Alexander Stolin	*The classification of quantum groups and Galois cohomology*

Abstracts for Tuesday June 13

Self-similarity for τ-groups

Olivier Mathieu (Lyon)

Let Γ be a group, let Θ be a subgroup and let $f : \Theta \to \Gamma$ be a morphism. The *f-core* of Θ is the biggest subgroup $C \subset \Theta$ such that C is normal in Γ and $f(C) \subset C$. The pair (f, Θ) is called a *similar data* for Γ if Θ and $f(\Theta)$ have finite index, if Ker$f$ is finite and if the f-core of Θ is trivial. The group Γ is called *self-similar* if it admits a similar data (f, Θ). A self-similar group acts faithfully on a tree. S. Sidki raised the question if any τ-group Γ is self-similar (a *τ-group* is a finitely generated torsion-free nilpotent group).

Let Γ be a τ-group. There is a unique simply connected connected nilpotent Lie group N such that Γ embeds in N as a cocompact lattice. Let $\mathfrak{n}_{\mathbb{R}}$ be the Lie algebra of N, let $\mathfrak{n}_{\mathbb{C}} = \mathbb{C} \otimes_{\mathbb{R}} \mathfrak{n}_{\mathbb{R}}$ and let $\mathfrak{z}_{\mathbb{C}}$ be the center of $\mathfrak{n}_{\mathbb{C}}$. In the talk, we will explain the following result:

Theorem: *The group Γ is self-similar iff the Lie algebra $\mathfrak{n}_{\mathbb{C}}$ admits a $\mathbb{Z}$-grading $\mathfrak{n}_{\mathbb{C}} = \oplus_{i \in \mathbb{Z}} \mathfrak{n}^i_{\mathbb{C}}$ such that $\mathfrak{n}^0_{\mathbb{C}} \cap \mathfrak{z}_{\mathbb{C}} = 0$.*

Leavitt path algebras and graph C*-algebras associated to separated graphs

Pere Ara (Barcelona)

A *separated graph* is a pair (E, C) consisting of a directed graph E and a set $C = \bigsqcup_{v \in E^0} C_v$, where each C_v is a partition of the set of edges whose terminal vertex is v. Leavitt path algebras $L_K(E, C)$ of separated graphs have been recently defined by Goodearl and the presenter [2]. They allow to incorporate the classical Leavitt algebras of any type (m, n) into the theory of graph algebras. Thanks to seminal work by George Bergman, it is possible to explicitly compute the monoids $\mathcal{V}(L_K(E, C))$ of finitely generated projective modules over these algebras.

Let (E, C) be a finite bipartite separated graph. In [1], a suitable quotient $L^{\mathrm{ab}}(E, C)$ of $L_K(E, C)$ is introduced, in such a way that the new algebra $L^{\mathrm{ab}}(E, C)$ can be expressed as a *partial crossed product* $C_K(\Omega(E, C)) \rtimes_{\alpha^*} \mathbb{F}$, where $\mathbb{F}$ is a free group and $\Omega(E, C)$ is a metrizable totally disconnected compact space, $C_K(\Omega(E, C))$ stands for the algebra of K-valued locally constant functions on $\Omega(E, C)$, and α is a partial action of $\mathbb{F}$ on $\Omega(E, C)$. Corresponding constructions with C*-algebras are also considered in [1].

I will review these constructions and will show an application of them to a problem on paradoxical decompositions, obtained in [1]. Following [3], I will also characterize the lattice of induced ideals of $L^{\mathrm{ab}}(E, C)$ in terms of certain subsets of vertices of an infinite separated graph (F_∞, D^∞), termed the *separated Bratteli diagram* of (E, C). Various examples will be considered.

References

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[3] P. Ara, M. Lolk, *Convex subshifts, separated Bratteli diagrams, and ideal structure of tame separated graph algebras*. Preprint.

Decomposability of Leavitt path algebras

Mercedes Siles (Mlaga)

The aim of this talk is to explain the characterization of undecomposable Leavitt path algebras of arbitrary graphs. A Leavitt path algebra (LPA) is said to be decomposable if it can be written as the direct sum of some of its ideals. Otherwise it is said to be undecomposable. There is a wide range of LPA which are not decomposable (for example, the simple or the graded simple) or such that their decompostions are known (as the locally artinian or the locally noetherian). It happens that the decomposability can be expressed in terms of central idempotents.

Key tools in our work will be the Steinberg algebras, as well as the structure of the center of a Leavitt path algebra.

We will also give a characterization of undecomposable Leavitt path algebras in terms of their underlying graph.

Abstracts for Wednesday June 14

On The classification of four-dimensional unital division algebras

Ernst Dieterich (Uppsala)

In their recent paper [1], M. Bani-Ata and his coauthors construct an exhaustive list for the class of all 4-dimensional unital division algebras over finite fields of odd order, whose left nucleus is not minimal and whose automorphism group contains Klein's four-group V.

In my talk, I will generalize the approach of [1] towards all algebras of the above mentioned type, but now admitting arbitrary fields k of characteristic not 2 as ground fields. This generalized approach evolves into unified constructive descriptions, in terms of group actions, of groupoids formed by 4-dimensional unital division k-algebras, that hitherto either were inaccessible (in the non-associative case), or treated by diverse other methods (in the associative case).

Among the applications, we prove that the 4-dimensional Hurwitz division k-algebras coincide with the 4-dimensional central skew fields over k whose automorphism group contains V, we classify all 4-dimensional unital division algebras over real closed fields, whose right nucleus is not minimal and whose automorphism group contains V, and in the finite field case we refine the Main Theorem of [1] to a classification even of the algebras studied there.

References

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On the classification of four-dimensional unital rational division algebras

Gustav Hammarhjelm (Uppsala)

In recent work by Ernst Dieterich, the category $\mathfrak{D}_k$ of four-dimensional unital division algebras whose right nucleus is non-trivial and whose automorphism group contains Klein's four group is studied over a general ground field k whose characteristic is not 2. In particular, the objects in $\mathfrak{D}_k$ are exhaustively constructed from parameters in k^3 and explicit k-dependent isomorphism conditions for the constructed objects are found.

In my talk, I will specialize the above situation to $k = \mathbb{Q}$ and present results towards a classification of $\mathfrak{D}_{\mathbb{Q}}$ obtained using the above findings. These results include explicit parametrized families of non-associative objects and a list that exhausts the subcategory of central skew fields. The redundance of the exhausting list will be discussed. Connections to classical results of elementary number theory that appeared over the course of this work will also be shown.

Local orders and Lesieur Croisot elements in Jordan pairs

Fernando Montaner (Zaragoza)

We report on joint ongoing research with Irene Paniello on pairs of quotients of Jordan pairs. Inspired by previous work on Jordan algebras by Tocón and Montaner, we introduce and study the notion of Lesieur Croisot element (LC-element, for short) of a Jordan pair and prove that the set of LC-elements forms an ideal $LC(V)$ in any nondegenerate Jordan pair V. We also introduce a notion

of Jordan Pair of quotients of a Jordan pair inspired on the definition of Johnson's ring of quotients as adapted to Jordan algebras by the authors, together with the notion of strong nonsingularity, also introduced by the authors for algebras, and prove that If a nondegenerate Jordan pair equals its LC-ideal, then it is strongly nonsingular.

Lie algebroids arising from infinite dimensional Lie theory

Arturo Pianzola (Edmonton)

A classical construction of Atiyah assigns to a (real or complex) Lie group G and principal G-bundle P over a manifold M, a Lie algebroid over M. The spirit behind our work is to put this work within an algebraic context, replace M by a scheme X and G by a "simple" reductive group scheme over X (in the sense of Demazure-Grothendieck) that arise naturally with an attached torsor (which plays the role of P) in the study of Extended Affine Lie Algebras. Lie algebroids in an algebraic sense were also considered by Beilinson and Bernstein. We will explain how the present work relates to theirs. This is joint work with J. Kuttler and F. Quallbrunn.

Abstracts for Thursday June 15

Nonassociative algebras of cubic forms

Vladimir Tkachev (Linköping)

In my talk I will explain how certain classes of metrized nonassociative algebras (NA) appear in three geometrical contexts: minimal cones, isoparametric hypersurfaces and eiconal equations. While these three problems have different background, the corresponding NA algebras have many common remarkable properties. For instance, they have natural hidden division algebra and Jordan algebras structures. We review some recent classification results.

Fundamental theorems of Doi-Hopf modules in a non-associative setting

Ramón González (Vigo)

Let H be a Hopf algebra. The notion of Hopf module was introduced by Larson and Sweedler [11]. Later, Doi [7] gave a generalization by considering a right H-comodule B. He introduced the category of (H, B)-Hopf modules (also called Doi-Hopf modules) and showed that, if M is an object in this category, there is an isomorphism of (H, B)-Hopf modules between M and the tensor product $M^{coH} \otimes_{B^{coH}} B$, where M^{coH} denotes the subobject of coinvariants of M. Also, in a work joint with Takeuchi [8], was proved the strong structure theorem for (H, B)-Hopf modules, that establishes a categorical equivalence between the category of (H, B)-Hopf modules and the one of right B^{coH}-modules.

On the other hand, in the latest years, many interesting generalizations of Hopf algebras have been developed. In particular, by weakening the conditions about (co)unit, Bhm, Nill and Szlachanyi introduced [5] the notion of weak Hopf algebra which has been very useful in order to study quantum group theory and operator algebras. More recently, by removing the associativity condition, Klim and Majid [10] introduced the notion of Hopf quasigroup in order to understand the structure and relevant properties of the algebraic 7-sphere. This notion arises naturally related with other structures in various non associative contexts, and relevant examples are the enveloping algebra U(L) of a Malcev algebra, as well as the notion of quasigroup algebra RL of an I.P. loop L.

It sounds natural to get the well-known results about Hopf algebras in these new settings, and many authors fit this program. In particular, the classical results in the context of weak Hopf algebras were obtained in the nineties of the last century [5], [9], [12] . Taking into account the novelty of the notion of Hopf quasigroup, only a few relevant results have been today obtained [6].

But a question naturally arises: Is it possible to introduce a new notion that encompass weak Hopf algebras and Hopf quasigroups? If true, any result obtained in this way led automatically to a double generalization, recalling the well-known results in the weak setting and showing the way to get the ones for the non associative context. Fortunately, we got a positive answer to this question by weakening the (co)unitality and associativity conditions. The new notion, called weak Hopf quasigroup, was developed in [1] (see also [2]). A family of non trivial examples of this algebraic structure can be obtained by working with bigroupoids, i. e., bicategories where every 1-cell is an equivalence and every 2-cell is an isomorphism.

Obviusly, in the path to get the classical results for this new structure, the fundamental theorem and the categorical equivalence are the first targets. The aim of this talk is to show that it is possible to obtain this fundamental theorem and the associated categorical equivalence for weak Hopf quasigroups. The more relevant results presented in the talk are contained in [4], where for a

weak Hopf quasigroup H and a right H-comodule magma B with a multiplicative total integral h, we introduce the notion of weak non-associative Doi-Hopf module and give the Fundamental Theorem of Hopf modules in this setting. Moreover, we establish a categorical equivalence between the category of strong (H, B, h)-Hopf modules and the category of right B^{coH}-modules. As particular instances we obtain the Fundamental theorem and the categorical equivalences for Hopf algebras, weak Hopf algebras, Hopf quasigroups and classical Hopf modules associated to weak Hopf quasigroups (see [3]).

References

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Graded and conformal modules over superconformal algebras

Consuelo Martínez (Oviedo)

The aim of this talk is to present the last advances, made together with O. Mathieu and E. Zelmanov, in the study of graded modules over Superconformal algebras. We will explain also a procedure that allows to relate graded modules an conformal modules, in a new approach to Superconformal algebras.

The classification of quantum groups and Galois cohomology

Alexander Stolin (Göteborg)

It is known that quantum groups can be understood via their Lie bialgebras. This motivates the classification of such bialgebras over fields of characteristic zero. I will explain this problem and an approach via Galois cohomology, obtained in recent joint work with A. Pianzola.
